Pseudo-reference for Motion Transfer based on Autonomous Control System with an Orbit Attractor

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Abstract-Skill or motion transfer from someone to other persons is always required for dance teaching, rehabilitation, sports and so on. For teaching or transfer of motion, a dance notation and display of sequence of motion are often used effectively. However, these contain only the instantaneous postures of motion or professional subjective sense, it is not easy for beginners to understand the dynamical knack of motion. In this paper, we propose "pseudo-reference" that is a virtual target posture in the human motion. The pseudoreference is developed based on the modeling of the human motion by the attractor design method which realizes an autonomous system. By comparing the nonlinear controller with conventional control systems (1DOF and 2DOF control system), the pseudo-reference is obtained as an embodiment of the implicit reference. The proposed method is performed with the inverted pendulum system and the tap dancing robot, and the validity and feasibility for motion transfer of the pseudoreference are evaluated.

I. INTRODUCTION

For human-human motion transfer, time-sequence posture variations will be utilized. Dance notation was developed to hand down a traditional dance to posterity. It contains not only special symbols that represent how to dance but also some complements by language. However the dance notation is difficult to understand for us because it is for dance artists who acquaint themselves with terpsichorean art. On the other hand, the time-sequence posture of the long jump is illustrated in the textbook of gymnastics as shown in figure 1. Because it represents only the kinematics of human body, some complements are used to explain the dynamical characteristic of the motion, for example at (a) in figure 1, it may be written 'Jump like running up the stairs' or 'Put your head forward' at (b). We will be able to imagine the corresponding postures as shown in the figure, but it is not easy for beginners because the compliments contain the athlete's instinct. The instinct frequently represents knack of motion which is an important factor to make efficient motion transfer. This concept is similar to human knowledge which consists of explicit and implicit knowledge[1]. For

smooth communication, the appropriate representation of implicit knowledge plays an important role[2]. The kinematic postures and the athlete's instinct analogizes with explicit and implicit knowledge respectively. Therefore the embodiment of the athlete's instinct leads the effective motion transfer.



Fig. 1. The sequence posture of long jump motion

Some results with the similar concept have been reported for robot control. Hasegawa et al.[3] and Cortesao et al.[4] divide the human skill of peg-in-hole into three motions (i) coming closer to the surface, (ii) rotation near the surface and (iii) arrangement to the hole, and realize this task by the robot. Ralph et al.[5] discuss on the human grip motion which consists of rising up, taking down, twisting, griping and letting go. Hirana et al.[6] segment the human motion into motion elements by means of Hidden Markov Model. Dordevic et al. [7] define the human skill from motion elements by learning expert motions. These methods focus on the representative motions to execute the given tasks effectively. Kuniyoshi et al.[8] propose a knack of robot motion in rolling and rising motion from a lot of measured motion patterns. Kawamura et al.[9] focus on the turning points of rotation, velocity and acceleration in the motion data, and defines another knack of motions. These methods give important key flame of motion from dynamical point of view. The obtained knack is selected from the explicit measured posture, however, it is difficult to embody the intuitive abstraction of artists or athletes, which will be an implicit motion.

The purpose of this paper is to embody the intuitive abstraction by the postures or images from dynamical point of view. To realize the embodiment of the implicit motion, the following strategy is employed in this paper. Firstly, (i) an autonomous control system, which is the attractor design method for robot control[10], is employed, because we assume that the autonomous control system is a model of the autonomous motion of human. This robot control method designs a nonlinear controller h(x) that makes the state variable x of the robot entrain to a specified orbit, and

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Fig. 2. Autonomous control system

realizes the autonomous control system as shown in figure 2. Based on the attractor design method, (ii) embodiment of the virtual reference posture including velocity information as pseudo-reference is realized by changing the controller formulation comparing to the classical control theory. The pseudo-reference does not coincide to the explicit motion data but realizes the target posture from dynamical point of view. And (iii) the effectiveness of the proposed method is evaluated by the simulations using inverted pendulum system (linear system for simplicity) and tap dancing robot (nonlinear system).

II. MODELING OF THE HUMAN MOTION BY AUTONOMOUS CONTROL SYSTEM

A. Autonomous control system design based on an orbit attractor

In reference [10], the autonomous control system design method based on orbit attractor is proposed. In this section, the controller design method is summarized. Consider the robot body dynamics represented by the following difference equation in discrete time domain;

$$x[k+1] = f(x[k]) + g(x[k], u[k])$$
(1)

where x[k] is a state variable, u[k] is an input of dynamics with a time stamp k. The controller is designed by the nonlinear function of x as follow;

$$u[k] = h(x[k]) \tag{2}$$

so that x[k] is entrained to a specified closed curved line Ξ ;

$$\Xi = \begin{bmatrix} \xi_1 & \xi_2 & \cdots & \xi_N \end{bmatrix}, (\xi_{N+1} = \xi_1)$$
(3)

in the state space, which means Ξ is an attractor of the closed loop system. Here we assume that Ξ is realizable, which means there exists the input sequence that realizes the motion Ξ to the dynamics. In fact, we design the nonlinear function by polynomial of ℓ -th order power of x as follow;

$$u[k] = \Theta\phi(x[k]) \tag{4}$$

where Θ is a coefficient matrix of polynomial and $\phi(x)$ expands the state vector x to the power vector of x. For example, $x \in \mathbb{R}^2$ and $\ell = 2$ cause ϕ as;

$$\phi(x) = \begin{bmatrix} 1 & x_1 & x_2 & x_1^2 & x_1x_2 & x_2^2 \end{bmatrix}^T$$
(5)

$$x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \tag{6}$$

By obtaining the realizable sets of (x[k], u[k]) for the motion, Θ is designed by functional approximation to minimize the following cost function J_{Θ} ;

$$\Theta = \arg\min_{\Theta} J_{\Theta} \tag{7}$$

$$J_{\Theta} = \sum_{k} \|u[k] - \Theta \phi(x[k])\|^2 \tag{8}$$

In the following section, the controller is represented as h(x) for simplicity.

B. Modeling of the autonomous motion

By using the attractor design method, we obtain the autonomous control system represented by figure 2, and the robot motion is realized autonomously without the explicit reference of the motion, which means the modeling of the autonomous human motion corresponds to the controller design. The design of the controller requires the body dynamics in equation (1) and Ξ in equation (3) which will be obtained from the human motion capture and inverse dynamics problem. Though the controlled system in figure 2 does not require the explicit reference, the controller parameter has the information of the emerged motion. By changing the controller formulation, the implicit reference is obtained as pseudo-reference.

III. DESIGN OF PSEUDO-REFERENCE

In this section, the pseudo-reference is designed by comparing the autonomous control system in figure 2 and the classical control methods. By the attractor design, the state variable x[k] converges to Ξ at $k \to \infty$. On the other hand, there are two methods that realize $x = \xi$. One is two degree of freedom model matching control system [11] shown in figure 3, where P is the controlled plant, P_m^{-1} is the inverse dynamical model of P, K is the feedback controller that stabilizes P and u is the input signal. Because the transfer



Fig. 3. Two DOF model matching control system

function from the input to the output is 1 with the assumption $P_m = P$, the state variable x converges to ξ at $k \to \infty$. In this system, the input u is obtained by;

$$u = P_m^{-1}\xi + K(\xi - x)$$
(9)

The other is one degree of freedom control system shown in figure 4, where K is the same feedback controller in figure 3, and x^{ref} means reference motion pattern. By setting x^{ref} as;

$$x^{ref} = \frac{1 + PK}{PK} \xi \tag{10}$$



Fig. 4. One DOF control system

the state variable x converges to ξ at $k \to \infty$. In this system, the input u is represented by;

$$u = K(x^{ref} - x) \tag{11}$$

On the other hand, the Taylor expansion of equation (2) around x using $\xi = x + \delta$ gives the following equation by neglecting more than second order terms assuming $\delta \ll 1$;

$$u = h(\xi) - \frac{\partial h(x)}{\partial x}(\xi - x)$$
(12)

By comparing equation (9) and (12), because the first term is concerned to ξ and the second term is concerned to $\xi - x$, we can regard K as;

$$K = -\frac{\partial h(x)}{\partial x} \tag{13}$$

This equation means that controller K is nonlinear function of x to realize the autonomous system. By substituting equation (13) into (11), the input u of the 1DOF feedback system is represented as;

$$u = -\frac{\partial h(x)}{\partial x} (x^{ref} - x) \tag{14}$$

By considering that the inputs u are same in the equation (2) and (14), the following equation is satisfied;

$$h(x) = -\frac{\partial h(x)}{\partial x}(x^{ref} - x)$$
(15)

and by solving the equation (15) for x^{ref} , we obtain;

$$x^{ref} = -\left(\frac{\partial h(x)}{\partial x}\right)^{\#} h(x) + x + \left(\frac{\partial h(x)}{\partial x}\right)^{\perp} \alpha$$
(16)

where $(\cdot)^{\#}$ means the Moore and Penrose pseudo-inverse, $(\cdot)^{\perp}$ means basis of null space and $(\cdot)^{\perp}\alpha$ means an arbitrary vector that belongs to the null space. We call x^{ref} as pseudo-reference of the autonomous system. By using the pseudo-reference, the feedback system in figure 2 is rewritten as figure 5. This feedback system means that the implicit reference x^{ref} is obtained based on the current state variable in the controller h(x) and the input signal is calculated based on the nonlinear feedback controller and difference between the reference x^{ref} and the state variable x. Here we remark that x^{ref} does not always coincide to ξ because it is obtained as a virtual reference from the dynamical point of view.



Fig. 5. Robot control system using the pseudo-reference



Fig. 6. Inverted pendulum system

IV. PSEUDO REFERENCE IN INVERTED PENDULUM SYSTEM

A. Modeling of the inverted pendulum motion

In this section, the pseudo-reference is calculated from the inverted pendulum motion. The model of an inverted pendulum system is shown in figure 6. The state variable x consists of rotational angle of pendulum θ , its angular velocity $\dot{\theta}$, cart position y and its velocity \dot{y} as follow;

$$x = \begin{bmatrix} \theta & \dot{\theta} & y & \dot{y} \end{bmatrix}^T \in R^4$$
(17)

The inverted pendulum motion is realized by the feedback system in figure 4 where K is designed by linear quadratic regulator and x^{ref} is selected appropriately. The realized motion is represented in figure 7. Though the state space of this system is four dimensional space, only the three dimensional state variables are shown in figure 7-(a) using θ , $\dot{\theta}$ and y. Figure 7-(b) shows the one cycle motion of the inverted pendulum. By setting the realized motion as Ξ in equation (3), the controller h(x) is designed and the inverted pendulum motion is modeled by an orbit attractor as shown in figure 8. This is realized based on the feedback system in figure 2.

B. Design of pseudo-reference

Because x^{ref} in the equation (16) is not uniquely decided, x^{ref} is designed to minimize the following cost function J_{ip} ;

$$J_{ip} = w_1 \left\| x^{ref} - x^{ref}_{pre} \right\|^2 + w_2 \left\| \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x^{ref} \right\|^2$$
(18)



Fig. 7. Original motion of the inverted pendulum system



Fig. 8. Motion of the inverted pendulum system model by the autonomous control system

where x_{pre}^{ref} is the previous pseudo-reference of x^{ref} at k-1and w_1 , w_2 are weighting factors. The first term makes distance between x_{pre}^{ref} and x^{ref} smaller to avoid radical variation of x^{ref} . The second term aims at obtaining the pseudo-reference by the position change of the cart. Figure 9 shows the obtained pseudo-reference. Figure 9-(a) represents the locus of the obtained pseudo-reference in the state space. Figure 9-(b) embodies the pseudo-reference as the posture of the inverted pendulum system.



Fig. 9. Pseudo-reference of the inverted pendulum motion

The pseudo-reference of inverted pendulum system often stops at two positions. On the other hand, the inverted pendulum motion in figure 7 is obtained by using x^{ref} as shown in figure 10. The step references $y^{ref} = \pm 0.7$ is utilized which stops in the two positions. From these results, it is said that

- The inverted pendulum motion in figure 7 is modeled as shown in figure 8 by using the autonomous control system.
- 2) The obtained pseudo-reference generates the implicit



Fig. 10. Explicit reference for the original motion

reference as shown in figure 9 that is same as the explicit reference in figure 10 that is utilized to generate the original motion in figure 7.

These results show the adequateness of the pseudoreference.

V. PSEUDO-REFERENCE IN TAP DANCING MOTION

A. Generation and modeling of the original motion

In the previous section, the pseudo-reference is applied to the inverted pendulum system. Because the linearized system is utilized, it is possible to design a stabilizing controller in figure 4 and to set the explicit reference x^{ref} to obtain the original motion. In this section, the pseudo-reference is applied to the tap dancing robot which is a nonlinear system and an explicit reference is difficult to be set. The tap dancing robot is shown in figure 11-(a) and its dynamical model is shown in figure 11-(b). This robot has been developed in [10]. It steps continuously by changing the grounding foot and be stabilized by shaking the head. The input is torque



Fig. 11. (a) Tap dancing robot [10] and (b) its dynamical model

u and the state variable *x* consists of lower body rotational angle θ_1 , its angular velocity $\dot{\theta}_1$, head rotational angle θ_2 and its angular velocity $\dot{\theta}_2$ as follow;

$$x = \begin{bmatrix} \theta_1 & \dot{\theta_1} & \theta_2 & \dot{\theta_2} \end{bmatrix}^T \in \mathbb{R}^4$$
(19)

We assume that the impacts of foot to the grounding is completely inelastic collision. Detail on the motion equations of the tap dancing robot is written in [10]. For this robot, the original motion Ξ in equation (3) is generated by giving



Fig. 12. Reference for θ_2

the following torque command with step input as shown in figure 12.

$$u(t) = \begin{cases} K_{\theta_2}(A - \theta_2), & nF \le t < (n + \frac{1}{2})F \\ K_{\theta_2}(-A - \theta_2), & (n + \frac{1}{2})F \le t < (n + 1)F \\ (n = 0, 1, 2 \cdots) & (20) \end{cases}$$

The notation means that K_{θ_2} is PD feedback controller, A is an amplitude of step signal, F defines the frequency of the tap dancing. The robot can move a dynamic stepping motion of full body. By using the torque command in figure 12, the robot temporarily realizes the tap dancing motion as shown in figure 13, however it is unstable because the rotational angle of the body θ_1 is not used for feedback stabilization. By clipping one cycle motion, the Ξ is obtained and this



Fig. 13. Original motion of the tap dancing robot

motion is modeled by the attractor design method.

B. Pseudo reference of tap dancing motion

From the equation (16), x^{ref} is not uniquely decided too. For the tap dancing motion, x^{ref} is designed to minimize the following cost function J_{td} ;

$$J_{\rm td} = w_3 \left\| x - x^{ref} \right\|^2 \tag{21}$$

where x is the state variable at the moment and w_3 is a weighting factor. This term makes distance between x and x^{ref} become smaller to avoid radical variation of x^{ref} in

the dynamical motion. Figure 14-(a) shows the locus of the state variable in the modeled tap dancing motion with the solid line and the pseudo-reference with the dots respectively. Figure 14-(b) is the projection of figure 14-(a) on θ_1 - θ_2 plane. Numbers in figure 14-(a,b) indicate motion sequence and corresponding postures and pseudo-references are also shown by solid line and dashed line respectively in figure 14-(b). The state variables and their pseudo-references are



Fig. 14. Pseudo-reference of the tap dancing motion

almost same in the half of the tap dancing motion which means that there are zero input of the control system in figure 5. On the other hand, pseudo-references are apart from the orbit of state variables at the point of dashed arrows. In these time, the control system generates input torque to continue the motion by changing the grounding foot. The control input is not always necessary but the timing of the energy injection is important to continue the tap dancing motion. The pseudo-reference gives us its timing and amplitude. Another characteristics point of pseudo-reference is that the pseudo-reference at dashed arrows moves to the outside of the orbit and keeps a step ahead of the state variable while it going to opposite direction. This accrues by nonholonomic characteristics of the tap dancing robot and is regarded as implicit knowledge to continue the dynamic motion.

C. Robot control with the pseudo-reference

To evaluate possibility of the motion transfer, the tap dancing robot is controlled based on the feedback system shown in figure 15. In this system, the feedback controller is same as in figure 5, however the pseudo-reference is not



Fig. 15. Robot control system with the explicit reference obtained from the pseudo-reference

online designed but obtained from figure 14 previously which means that x^{ref} is an embodiment of the implicit reference of other robot and this robot realizes its motion based on the displayed posture. Figure 16-(a) shows the locus of the state variables in regenerated motion. Numbers of figure 16-(a) indicate the motion sequence, and corresponding postures and pseudo-references are also shown by solid line and dashed line respectively in figure 16-(b).



Fig. 16. Robot control simulation of the tap dancing robot with the explicit pseudo-reference

- 1) In this simulation, the pseudo-reference is clipped to one cycle data, and x^{ref} assigns it repetitively.
- 2) x^{ref} is reset to the nearest clipped pseudo-reference from x at every cycle to avoid phase shifting because the cycle times of the tap dancing motion are always different.

The robot can realize the original motion continuously and successfully by using the pseudo-reference with implicit knowledge of tap dancing motion. This result shows that the pseudo-reference, which embodies the implicit reference, is useful for motion transfer as the explicit reference.

VI. CONCLUSIONS

In this paper, we model the autonomous motion by the autonomous control system using attractor design, and the pseudo-reference is introduced to embody the implicit reference for motion transfer. The results of this paper are summarized as follows;

- By comparing the autonomous system and the linear control systems, the design method of pseudoreference is proposed using the controller with an orbit attractor.
- The proposed method is evaluated by the inverted pendulum system. The obtained pseudo-reference represents the original explicit reference.
- 3) The proposed method is also evaluated by the stepping motion of tap dancing robot. From the results of the simulation, the timing of the energy injection to continue the motion is discussed.
- 4) The robot regenerates the original motion by using the pseudo-reference as the explicit reference. This results shows that the pseudo-reference can realize the motion transfer.

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