# Design of the Continuous Symbol Space for the Intelligent Robots using the Dynamics-based Information Processing 

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#### Abstract

In this paper, we design the continuous symbol space using the dynamics-based information processing system. One point in the symbol space decides a vector field in the motion space that generates the cyclic motion and the continuous motion transition of the robots. Because the motion of the state vector in the symbol space is defined by a dynamical system, the spatial and temporal continuous information processing system is realized. Keywords: brain-like information processing, dynamical system, continuous system, symbol space


## 1 Introduction

So far, for the intelligent robots, the information processing system that generates the robot motion has been represented by the discrete time event system based on the artificial intelligence. In this system, the motions are labeled by the symbols such as "walk", "run" and so on, and each symbol is corresponding to the motion pattern trajectory and the robust feedback controller that stabilize the robot. Because the discrete time event system eliminates the temporal continuity, the motion change and the motion transition are independent to the real time, and executed based on the sensor signal or the somatic sensation of the robot. The timing of these events is given by the designer directly or by the cost function for optimality. The motion pattern trajectory is fixed and only the designed pattern is generated. The start posture and the final posture are given, that yields the discontinuous motion transition. This is due to the elimination of the spatial continuity in the motion generation system. For the smooth motion and the motion transition of the robot in the real world, the temporally and spatially continuous information processing system and motion generation system are necessary.

On the other hand, in the biological information processing, the nonlinear dynamical phenomena are found. Free-
man showed the nonlinear dynamical phenomena in rabbits' olfactory perception $[1,2,3]$. For the known smell, the order is seen and for the unknown smell, the chaotic phenomenon is seen. Tsuda showed the importance of the chaotic dynamics for learning and intelligence, and calls the phenomenon such that the human brain transits from one attractor to another as 'chaotic itinerancy' [4]. Because the dynamics is the temporally and spatially continuous system, the effectiveness of dynamics for the tool of the information processing in the real world is paid much attention. It is known that the CPG (Central pattern generator) has the entrainment phenomenon in the natural frequency. The walk motion generation of the quadruped locomotion robot was realized based on the dynamic interaction with the environment [5] and the rhythmical motion was realized [6] using the CPG. Ijspeert developed the movement imitation of the humanoid robot using the CPG based dynamics [7].

On the other hand, we have developed the design method of the dynamics that has an attractor on the closed curved line in the $N$ dimensional space, and proposed the dynamics-based information processing system that generated the humanoid whole body motion [8]. By changing the configuration of the dynamics, the smooth motion transition is realized. This system uses the temporal and spatial continuous characteristic of the dynamics.

In this paper, based on the dynamics-based information processing system, we develop the design method of the continuous symbol space that represents the humanoid motion, and the motion reduction method for the design of the motion space using the nonlinear mapping function. One point in the symbol space defines one dynamics in the motion space, and the motion of the state vector in the motion space yields the motion trajectory of the humanoid motion, which means the point in the symbol space represents the symbol of the motion. Any points in the symbol space correspond to the dynamics in the motion space, which means the symbol space is spatially continuous. The motion of the state vector in the symbol space according to the dynamics yields the change of the
configuration of the dynamics in the motion space, which yields the temporally continuous motion generation and motion transition.

## 2 Dynamics-based information processing

In this section, we will explain about the dynamics-based information processing system[8]. Consider the cyclic whole body motion $M$ of the robot with $N$ joints. The posture (joint angles) $\theta[k]$ in time $k$ is represented by one point in $N$ dimensional joint space. The whole body motion $M$ consists of the sequence of $\theta[k]$ as follows,

$$
M=\left[\begin{array}{llll}
\theta[1] & \theta[2] & \cdots & \theta[m] \tag{1}
\end{array}\right]
$$

and it draws a closed curved line $C$ as shown in Figure 1, where $m$ means the number of data.


Figure 1: Robot posture, motion in the joint space

On the other hand, consider the discrete time dynamics as shown in the following difference equation.

$$
\begin{equation*}
\boldsymbol{x}[k+1]=\boldsymbol{x}[k]+\boldsymbol{f}(\boldsymbol{x}[k]) \tag{2}
\end{equation*}
$$

Suppose that this dynamics has an attractor on the closed curved line $C$, which means the state vector $\boldsymbol{x}[k]$ start from the initial value $\boldsymbol{x}_{0}$ converges to the following equation

$$
\begin{equation*}
\lim _{k \rightarrow \infty} x[k]=\theta\left[k+k_{0}\right] \tag{3}
\end{equation*}
$$

where $k_{0}$ depends on $\boldsymbol{x}_{0}$. In this case, the dynamics memorizes and reproduces the whole body motion $M$.

By defining $f(x[k])$ as the vector field in $\boldsymbol{x}$-space, the dynamics in equation (2) is designed by the polynomial
configuration using the least square method. The detail is omitted in this paper.

When the dynamics memorizes multi-motions $M_{1}, M_{2}$, $\cdots, \boldsymbol{f}(\boldsymbol{x}[k])$ in equation (2) is represented by the summation of the vector fields as follows.

$$
\begin{equation*}
\boldsymbol{f}(\boldsymbol{x}[k])=\sum_{i} w_{i} \boldsymbol{f}_{i}(\boldsymbol{x}[k]) \tag{4}
\end{equation*}
$$

By changing the weighting parameters $w_{i}$, the basin of the attractors are changed.

## 3 Motion reduction using the nonlinear mapping function

Because the robot such as a humanoid robot, has many joints, the dimension of $x[k]$ increases, and to design the dynamics costs longer time and requires larger computational power, the motion reduction method is required. Tatani[9] proposed the motion reduction method using the neural network based NLPCA method. In this method, the reduced motion cannot be specified. We proposed the motion reduction method using the principal component analysis based on the singular value decomposition[8]. Because this method uses the linear projection, the reduced dimension is not so small.

In this paper, we propose the motion reduction method using nonlinear mapping function. From the reduced state vector $\boldsymbol{x}[k] \in \boldsymbol{R}^{n}$, the joint angle $\theta[k] \in \boldsymbol{R}^{N}$ is generated by the following equation.

$$
\begin{equation*}
\theta[k]=F(x[k]) \tag{5}
\end{equation*}
$$

We obtain the common $F$ for all motions $M_{i}$ by using the nonlinear function with the polynomial configuration. For the motion $M_{i}$, the reduced closed curved line $C_{i}$ is calculated, and the dynamics in equation (2) is designed.

Consider the robot motion $M$ with $N$ joints represented by the following equation.

$$
M=\left[\begin{array}{llll}
\theta[1] & \theta[2] & \cdots & \theta[m] \tag{6}
\end{array}\right] \in \boldsymbol{R}^{N \times m}
$$

The reduced representation of $M$ is assume to be represented by $C$

$$
C=\left[\begin{array}{llll}
\boldsymbol{x}[1] & \boldsymbol{x}[2] & \cdots & \boldsymbol{x}[m] \tag{7}
\end{array}\right] \in \boldsymbol{R}^{n \times m}
$$

where $n<N$ is satisfied. We obtain the mapping function in equation (5) that calculates $\theta[k]$ from $\boldsymbol{x}[k]$. We set the configuration of $F(\boldsymbol{x}[k])$ as the polynomial function of $x_{i}(i=1,2, \cdots, n)$ that are the elements of $\boldsymbol{x}$. For example, when $n=2$, the $\ell$-th order polynomial function is represented by

$$
\begin{align*}
\theta & =A \xi  \tag{8}\\
A & =\left[\begin{array}{llllll}
a_{\ell 0} & a_{(\ell-1) 1} & a_{(\ell-1) 2} & \cdots & a_{01} & a_{00}
\end{array}\right]  \tag{9}\\
\xi & =\left[\begin{array}{llllll}
x_{1}^{\ell} & x_{1}^{(\ell-1)} x_{2} & x_{1}^{(\ell-2)} x_{2}^{2} & \cdots & x_{2} & 1
\end{array}\right]^{T} \tag{10}
\end{align*}
$$

When $C$ is given, $A$ is obtained by the solution of the least square method. In the following, the calculation algorithm for $F(\boldsymbol{x}[k])$ is illustrated.

Step1 From $\theta[k]$ in $M$, obtain the following matrix.

$$
\Theta=\left[\begin{array}{llll}
\theta[1] & \theta[2] & \cdots & \theta[m] \tag{11}
\end{array}\right]
$$

Define $\boldsymbol{x}[k]$ in the closed curved line $C$ that corresponds to the motion in the reduced space, and obtain the following matrix.

$$
\Xi=\left[\begin{array}{llll}
\xi[1] & \xi[2] & \cdots & \xi[m] \tag{12}
\end{array}\right]
$$

Any pattern of $\boldsymbol{x}[k],(k=1,2, \cdots, m)$ is available except self cross curves.

Step2 From $\Theta$ and $\Xi$, obtain $A$ as follows.

$$
\begin{equation*}
A=\Theta \Xi^{\#} \tag{13}
\end{equation*}
$$

where [.] ${ }^{\#}$ means the pseudo inverse matrix.
Step3 To improve the accuracy of $A \Xi$ and $\Theta$, we change $\boldsymbol{x}[k]$ as follows. Set the criterion function $J[k]$ as

$$
\begin{equation*}
J[k]=\frac{1}{2}\|\theta[k]-A \xi[k]\|^{2} \tag{14}
\end{equation*}
$$

and change $\boldsymbol{x}[k]$ as follows.

$$
\begin{align*}
\boldsymbol{x}[k] & =\boldsymbol{x}[k]-\frac{\partial J}{\partial \boldsymbol{x}} \delta  \tag{15}\\
\frac{\partial J}{\partial \boldsymbol{x}} & =\left(-A \frac{\partial \xi}{\partial \boldsymbol{x}}\right)^{T}(\theta[k]-A \xi[k]) \tag{16}
\end{align*}
$$

where $\delta$ is constant.
Step4 Iterate Step2 and Step3.

Figure 2 shows the example of the calculation of $C$ and $A$. The upper figure shows the initial pattern of $M \in$ $\boldsymbol{R}^{3}$ (dashed line) and obtained $\Theta=A \Xi$ (solid line), the lower figure shows the original $C \in \boldsymbol{R}^{2}$ (dashed line) and modified $C$ (solid line). The upper figure shows the joint space and the closed curved line shows the whole body motion. In the lower figure, the original $C$ is given by the circle, and after the iteration, the reduced motion is given by the dashed line that approximate $M$ more accurately.

## 4 Design of the continuous symbol space

### 4.1 Symbol space and motion space

In this section, we explain about the continuous symbol space that defines the dynamics in the motion space and the design method of the dynamics in the continuous symbol space. Figure 3 shows the concept of the continuous


Figure 2: Projection to reduced space
symbol space. One point in the symbol space defines the dynamics in the motion space. The state vector moves following the vector field of the dynamics. The state vector in the symbol space moves following the dynamics, which changes the configuration of the dynamics in the motion space, and the motion generation and transition of the robot is realized.


Figure 3: Symbol space and motion space

### 4.2 Design of the symbol space

In this section, we describe the design method of the symbol space and dynamics.

Step1 Consider the two motions $M_{1}$ and $M_{2}$ represented
by

$$
\begin{align*}
& M_{1}=\left[\begin{array}{llll}
\theta_{1}[1] & \theta_{1}[2] & \cdots & \theta_{1}[m]
\end{array}\right]  \tag{17}\\
& M_{2}=\left[\begin{array}{llll}
\theta_{2}[1] & \theta_{2}[2] & \cdots & \theta_{2}[m]
\end{array}\right] \tag{18}
\end{align*}
$$

Step2 Obtain some motions $M_{i}$ between $M_{1}$ and $M_{2}$. For example $M_{i}$ are calculated from the following equation.

$$
\begin{equation*}
M_{i}=\left(1-\alpha_{i}\right) M_{1}+\alpha_{i} M_{2}(i=3,4,5, \cdots) \tag{19}
\end{equation*}
$$

where, $0<\alpha_{i}<1$ is satisfied.
Step3 For the whole body motions $M_{i}$, obtain the reduced motion $C_{i}$

$$
C_{i}=\left[\begin{array}{llll}
\boldsymbol{x}_{i}[1] & \boldsymbol{x}_{i}[2] & \cdots & \boldsymbol{x}_{i}[m] \tag{20}
\end{array}\right]
$$

For all $i$ and $k$, the function $F(\cdot)$ satisfies the following equation.

$$
\begin{equation*}
\theta_{i}[k]=F\left(\boldsymbol{x}_{i}[k]\right) \tag{21}
\end{equation*}
$$

Step4 Design the dynamics with the polynomial configuration

$$
\begin{equation*}
\boldsymbol{\lambda}_{i}[k+1]=\boldsymbol{\lambda}_{i}[k]+\boldsymbol{g}_{i}\left(\boldsymbol{\lambda}_{i}[k]\right) \tag{22}
\end{equation*}
$$

that has an attractor on $C_{i}$. Where, $\boldsymbol{g}_{i}\left(\boldsymbol{\lambda}_{i}[k]\right)$ is represented by

$$
\begin{equation*}
\boldsymbol{g}\left(\boldsymbol{\lambda}_{i}[k]\right)=\Phi_{i} \xi\left(\boldsymbol{\lambda}_{i}[k]\right) \tag{23}
\end{equation*}
$$

$\Phi_{i}$ consists of the coefficients of the polynomial function same as $A$ shown in equation (8).
Step5 From $\Phi_{i}(i=1,2, \cdots p)$, obtain $\lambda$ and $\phi$ that satisfy the following equation.

$$
\left[\begin{array}{c}
\Phi_{1}  \tag{24}\\
\Phi_{2} \\
\vdots \\
\Phi_{p}
\end{array}\right]=\left[\begin{array}{cccc}
\lambda_{11} I & \lambda_{12} I & \cdots & \lambda_{1 q} I \\
\lambda_{21} I & \lambda_{22} I & \cdots & \lambda_{2 q} I \\
\vdots & \vdots & & \vdots \\
\lambda_{p 1} I & \lambda_{p 2} I & \cdots & \lambda_{p q} I
\end{array}\right]\left[\begin{array}{c}
\phi_{1} \\
\vdots \\
\phi_{p}
\end{array}\right]
$$

By defining $\Phi_{i}$

$$
\Phi_{i}=\left[\begin{array}{lll}
\Phi_{1 i}^{T} & \cdots & \Phi_{N i}^{T} \tag{25}
\end{array}\right]^{T}
$$

the singular value decomposition of $\boldsymbol{\Phi}$

$$
\begin{gather*}
\mathbf{\Phi}=\left[\begin{array}{cccc}
\Phi_{11} & \Phi_{21} & \cdots & \Phi_{N 1} \\
\Phi_{12} & \Phi_{22} & \cdots & \Phi_{N 2} \\
\vdots & \vdots & & \vdots \\
\Phi_{1 p} & \Phi_{2 p} & \cdots & \Phi_{N p}
\end{array}\right]  \tag{26}\\
\mathbf{\Phi}=\left[\begin{array}{ccc}
\lambda_{11} & \cdots & \lambda_{1 q} \\
\lambda_{21} & \cdots & \lambda_{2 q} \\
\vdots & & \vdots \\
\lambda_{p 1} & \cdots & \lambda_{p q}
\end{array}\right]\left[\begin{array}{c}
\phi_{1} \\
\vdots \\
\phi_{q}
\end{array}\right] \tag{27}
\end{gather*}
$$

gives the solution of $\lambda$ and $\phi$. Equation (24) means that the plurality of the dynamics $\boldsymbol{\Phi}_{i}$ that have the polynomial configuration are represented by the less number of the basis function $\phi_{j}$, which means the reduction of the functional space of the dynamics.


Figure 4: Connection of the symbol space and the motion space

Step6 The vector $\boldsymbol{\lambda}_{i}$

$$
\boldsymbol{\lambda}_{i}=\left[\begin{array}{lll}
\lambda_{i 1} & \cdots & \lambda_{i q} \tag{28}
\end{array}\right]^{T}
$$

represents a point in $q$-dimensional $\phi$-space. The vector $\boldsymbol{\lambda}_{i}$ defines $\Phi_{i}$ as follows,

$$
\Phi_{i}=\boldsymbol{\lambda}_{i}^{T}\left[\begin{array}{lll}
\phi_{1} & \cdots & \phi_{q} \tag{29}
\end{array}\right]^{T}
$$

that defines the dynamics in the motion space. This result shows the spatially continuous connection such that the state vector in the symbol space defines the motion space and the state vector in the motion space defines the robot motion. By defining the dynamics that decides the motion of the state vector $\boldsymbol{\lambda}_{i}$, the temporally continuous connection is realized.

The process of the information processing is represented in Figure 4. $\Phi$ is obtained by the summation of $\phi_{i}$ at the rate of $\lambda_{i}$. The dynamics defined by $\Phi$ yields the motion of $\boldsymbol{x}[k]$. By the mapping function $F$, the reduced motion $\boldsymbol{x}[k]$ is extracted to the joint angles of the robot
$\theta[k]$. Because the symbol space is continuous space and the dynamics in this space is continuous system, the continuous change of the vector field in the motion space and the continuous motion transition is realized.

## 5 Motion generation of the humanoid robot

In this section, we generate the whole body motion of the humanoid robot shown in Figure 5. This robot is HOAP-


Figure 5: Humanoid robot HOAP-1
1 produced by FUJITSU Co., and it has 20 joints. We design the walk motion $W_{w}$ and the squat motion $W_{s}$ that are shown in Figure 6. The upper figure shows the walk motion and the lower motion shows the squat motion. We


Figure 6: Motion of the humanoid robot
obtain 22 motions interpolating $M_{w}$ and $M_{s}$ as follows.

$$
\begin{align*}
M_{i} & =\left(1-\frac{i}{21}\right) M_{w}+\frac{i}{21} M_{s}  \tag{30}\\
i & =0,1, \cdots 21
\end{align*}
$$

Each motion is reduced to 3 dimensional motion $C_{i}$ and we obtain the function $F(\boldsymbol{x}[k])$ that extracts $\boldsymbol{x}[k]$ to the joint angles. And we design the dynamics that has an
attractor on $C_{i}$ as follows.

$$
\begin{equation*}
\boldsymbol{x}[k+1]=\boldsymbol{x}[k]+\boldsymbol{\Phi}_{i} \xi(\boldsymbol{x}[k]) \tag{31}
\end{equation*}
$$

Based on Step5, we obtain the 8 dimensional symbol space. The reduced motion $C_{i}$ correspond to $\boldsymbol{\lambda}_{i}(i=$ $0,1, \cdots 21)$. The state vector $\boldsymbol{\lambda}[k]$ in the symbol space moves according to the dynamics

$$
\begin{equation*}
\boldsymbol{\lambda}[k+1]=\boldsymbol{\lambda}[k]+\boldsymbol{g}(\boldsymbol{\lambda}[k]) \tag{32}
\end{equation*}
$$

that has an attractor on the opened curved line $\boldsymbol{\lambda}_{0} \rightarrow \boldsymbol{\lambda}_{1}$ $\rightarrow \cdots \lambda_{21}$. Figure 7 shows the generated humanoid whole body motion. The left hand side shows the motion of the dynamics in the symbol space. The symbol space is design as the 8 dimensional space, however only 3 dimensions are illustrated for simplicity. The middle shows the motion of the dynamics in the reduced motion space and the right hand side shows the motion of the humanoid robot. The point $\boldsymbol{\lambda}[k]$ in the symbol space defines the


Figure 7: Motion generation of the humanoid robot
generated motion and the point $\boldsymbol{x}[k]$ in the motion space defines the trajectory of the robot joints. The smooth motion transition is realized.

## 6 Conclusions

The results of this paper are as follows.

1. We design the continuous symbol space.
(a) One point in the symbol space defines the dynamics in the motion space, which means the point in the symbol space corresponds to the symbol of the motion.
(b) All the point in the symbol space define a robot motion, which means the symbol space is spatially continuous.
(c) The point in the symbol space moves according to the dynamics, which means the information processing is timely continuous.
2. We propose the motion reduction method using the nonlinear mapping function, and design the reduced motion space.
(a) One point in the motion space defines the posture of the robot and the motion of the state vector in the motion space yields the whole body motion.
(b) The dynamics in the motion space produces the trajectory of the joint angles.
(c) The configuration of the dynamics in the motion space is changed by the dynamics in the symbol space, which is temporally and spatially continuous.
3. We design the symbol space and the motion space that yields the humanoid robot motion with 20 degrees-of-freedom.

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