Controller Reduction for Pseudo-Reference in High-Degree of Freedom Control System

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ABSTRACT

Dance teaching, sports teaching and rehabilitation require instruction of motion from an expert to a beginner. So far, we have proposed "Pseudo-Reference" based on attractor design method. The pseudo-reference is a virtual reference which is derived from a controller of an autonomous control system and shows a target posture to execute the motion. However, for high-degree of freedom systems, because the controller has to be a high order function, it is not easy to be obtained. In this paper, we propose a controller reduction method for attractor design. Based on the correlation of motion data, the principal component analysis gives us appropriate low dimensional space of the motion. The effectiveness is evaluated by using the tap dancing robot, and pseudo-reference is applied to human motion.

Index Terms: I.2.9 [ARTIFICIAL INTELLIGENCE]: Robotics—Kinematics and dynamics; I.2.8 [ARTIFICIAL INTELLIGENCE]: Problem Solving, Control Methods, and Search—Control theory

1 INTRODUCTION

For human-human motion instruction, we sometimes use dance notation or time sequence posture variation. The dance notation was developed to hand down a traditional dance to posterity. However the dance notation is difficult to understand for beginners, because it is for experts who acquaint themselves with terpsichorean art. Moreover, for example, the time sequence posture of the long jump is illustrated in the textbook of gymnastics as shown in Figure 1. Because the figure represents only the kinematic information of the motion, some annotations are used to explain the dynamical characteristic of the motion. The instructor will say 'Jump like running up stairs' or 'Put your head forward'. We may imagine the corresponding postures, but it is difficult for beginners because the annotations include personal inspirations. The annotations frequently represent a knack of motion which is an important factor to make efficient

![Figure 1: The time sequence posture of long jump](image)

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1 INTRODUCTION

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![Figure 1: The time sequence posture of long jump](image)

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which realize $P$ is a inverse dynamical model of $x$ as follows;

$$x[k+1] = f(x[k]) + g(x[k], u[k])$$  \(1\)

where $x \in \mathbb{R}^n$ is a state variable, $u \in \mathbb{R}^m$ is an input at time stamp $k$. The controller is designed by a nonlinear function of $x$ as follows;

$$u[k] = h(x[k])$$  \(2\)

so that $x$ is entrained to a specified closed curved line $\Xi$ as;

$$\Xi = [\xi_1 \xi_2 \cdots \xi_N] \ (\xi_{N+1} = \xi_1)$$  \(3\)

$\Xi$ is assumed to be realizable which means there exists an input sequence that realizes motion $\Xi$ for the dynamics. The block diagram of the autonomous system using equation (1) and (2) is shown in Figure 2. The robot realizes the motion $\Xi$ without external input.

![Figure 2: Autonomous control system](image)

For a nonlinear function in equation (2), a $\ell$-th order polynomial of $x$ as;

$$u = a_0 + a_1 x + a_2 x^2 + \cdots + a_\ell x^\ell$$  \(4\)

$$\Theta = [a_0 a_1 \cdots a_\ell]$$  \(5\)

$$\phi(x) = [1 \ x^T \ x_1^T \ \cdots \ x_\ell^T]^T$$  \(6\)

is utilized. $a_i \ (i = 1, \cdots, \ell)$ and $\Theta$ are coefficient matrices of the polynomial function, and $\phi(x)$ expands $x$ to the power vector. For example, $x \in \mathbb{R}^3$ and $j = 2$ defines $x^j$ as;

$$x^2 = [x_1^2 \ x_1 x_2 \ x_1 x_3 \ x_2^2 \ x_2 x_3 \ x_3^2]^T$$  \(8\)

By setting realizable pairs of $(u_i, x_i)$, $\Theta$ is obtained by the least mean square approximation as;

$$\Theta = U \Phi^\#$$  \(9\)

$$U = [u_1 \ u_2 \ \cdots \ u_N]$$  \(10\)

$$\Phi = [\phi(x_1) \ \phi(x_2) \ \cdots \ \phi(x_N)]$$  \(11\)

where $[.]^\#$ means a pseudo inverse matrix defined by;

$$\Phi^\# = \Phi^T (\Phi \Phi^T)^{-1}$$  \(12\)

### 2.2 Pseudo-reference design

In this section, we summarize the pseudo-reference design method[6]. The closed loop system in Figure 2 does not have any external input. By using the decomposition of the controller, the system of Figure 2 is changed into that of Figure 3 which has a virtual reference $x^{ref}$ inside the controller. By considering the comparison between autonomous system and linear controlled systems, $h(x)$ is decomposed. The state variable $x$ converges to $\Xi$ by the attractor design as $k \to \infty$. On the other hand, there are two methods which realize $x = \xi$. One is a two degree of freedom control system (model matching) [14] as shown in Figure 4. $P$ is a plant, $P_m^{-1}$ is a inverse dynamical model of $P$, $K$ is a feedback controller that stabilizes $P$ and $u$ is an input to $P$. Because the transfer function from the input to the output of the closed loop system is equal to 1 with the assumption $P_m = P$, $x$ converges to $\xi$ as $k \to \infty$. In this system, $u$ is obtained by;

$$u = P_m^{-1} \xi + K (\xi - x)$$  \(13\)

The other method, which realizes $x = \xi$, is appropriate selection of reference in the one degree of freedom control system as shown in Figure 5. $K$ is the same feedback controller in Figure 4, and $x^{ref}$ is the reference motion pattern. By setting $x^{ref}$ as;

$$x^{ref} = \frac{1 + PK}{PK} \xi$$  \(14\)

$x$ converges to $\xi$ as $k \to \infty$. In this system, $u$ is represented as;

$$u = K (x^{ref} - x)$$  \(15\)

The first order Taylor expansion of equation (2) around $x$ using $\xi = x + \delta \ (\delta \ll 1)$ gives the following equation.

$$u = h(\xi) - \frac{\partial h(x)}{\partial x} (\xi - x)$$  \(16\)

$\partial h/\partial x$ is differential of $h(x)$ with respect to the each element of $x$. For example, $x \in \mathbb{R}^3$ defines $\partial h/\partial x$ as;

$$\frac{\partial h(x)}{\partial x} = \begin{bmatrix} \frac{\partial h(x)}{\partial x_1} \frac{\partial h(x)}{\partial x_2} \frac{\partial h(x)}{\partial x_3} \end{bmatrix}^T$$  \(17\)

By comparing equation (13) and (16), because the first term is concerned to $\xi$ and the second term is concerned to $\xi - x$, we can regard

![Figure 3: Robot control system using the pseudo-reference](image)

![Figure 4: Two DOF model matching control system](image)

![Figure 5: One DOF control system](image)
\[ K = -\frac{\partial h(x)}{\partial x} \]  

(18)

This equation means that to realize the autonomous system the controller \( K \) as a nonlinear function of \( x \). By substituting equation (18) into (15), the input \( u \) of the 1DOF feedback system is represented as;

\[ u = -\frac{\partial h(x)}{\partial x} (x^{ref} - x) \]  

(19)

By considering that the inputs \( x \) and \( u \) in equation (2) and (19) are the same, we obtain;

\[ x^{ref} = -\left( \frac{\partial h(x)}{\partial x} \right)^n h(x) + x + \left( \frac{\partial h(x)}{\partial x} \right)^{\perp} \alpha \]  

(20)

where \( [\cdot]^{\perp} \) means basis of null space and \( [\cdot]^{\perp} \alpha \) means an arbitrary vector that belongs to the null space. We call \( x^{ref} \) in equation (20) the pseudo-reference of the autonomous system. It is a virtual reference inside the controller, which means that an implicit reference is obtained based on the current state variable. Here we remark that \( x^{ref} \) does not always coincide with \( \xi \) because \( x^{ref} \) is obtained as a virtual reference from dynamical point of view.

### 3 CONTROLLER REDUCTION

#### 3.1 Controller reduction using principal component analysis

A larger \( \ell \) in equation (5) causes a larger number of terms in polynomial function. The number of terms \( L \) is calculated as follows;

\[ L = 1 + \sum_{i=1}^{\ell} n_i H_i = \sum_{i=0}^{\ell} \frac{(n + i - 1)!}{(n - 1)!i!} \]  

(21)

where \( n_i H_i \) is a repeated combination, \( n \) and \( \ell \) are the dimension of \( x \) and the power respectively. Table 1 shows the value of \( L \) with respect to \( n \) and \( \ell \). \( L \) increases rapidly according to the increase of

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \ell )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>\ldots</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td>20</td>
<td>35</td>
<td>56</td>
<td></td>
<td>286</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>35</td>
<td>70</td>
<td>126</td>
<td></td>
<td>1001</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>56</td>
<td>126</td>
<td>252</td>
<td></td>
<td>3003</td>
</tr>
<tr>
<td>\ldots</td>
<td></td>
<td>1330</td>
<td>7315</td>
<td>33649</td>
<td></td>
<td>10^7&gt;</td>
</tr>
</tbody>
</table>

\( n \) and \( \ell \). For example, in the case of the \( n = 18 \) which represents a planar human legged model, more than several thousand terms are required. Higher order dimension makes it difficult to design an autonomous system. Actually, calculation amount in polynomial function is as follows;

\[ c(n, \ell) = m \left[ \sum_{i=0}^{\ell} \left( \frac{(n + i - 1)!}{(n - 1)!i!} (i + 1) \right) \right] - 1 \]  

(22)

where \( m \) is the dimension of \( u \). Then by using the Stirling’s approximation, the order of \( c(n, \ell) \) is required more and more calculation cost as follows;

\[ O^{(c)} \approx O \left( m \sqrt{\frac{n + \ell - 1}{(n - 1)^{\ell}} \frac{(n + i - 1)!}{(n - 1)^{\ell}} (i + 1)} \right) \]  

(23)

To overcome this problem, we define a new controller formulation as;

\[ u = a_0 + a_1 x + a_2 x^2 + \cdots + a_\ell x^\ell \]  

(24)

\[ \hat{\phi}(x, Q) = h(x, Q) \]  

(25)

\[ \hat{\phi}(x, Q) = \left[ 1 \ x^T \ x^2 \ cdots x^{\ell} \right]^T \]  

(26)

where \( r < n \) and \( Q \in R^{r \times n} \) is a constant matrix. \( Q \) is obtained by principal component analysis of \( \Xi \) in (3) as follows;

\[ \Xi = USV^T \]  

(28)

\[ = \left[ U_1 \ U_2 \right] \left[ \begin{array}{cc} S_1 & 0 \\ 0 & S_2 \end{array} \right] \left[ \begin{array}{c} V_1^T \\ V_2^T \end{array} \right] \]  

(29)

\[ S_1 = \text{diag} \{ s_1, s_2, \cdots, s_r \} \]  

(30)

\[ S_2 = \text{diag} \{ s_{r+1}, s_{r+2}, \cdots, s_n \} \]  

(31)

By assuming \( s_r \gg s_{r+1} \), \( Q \) is obtained by;

\[ Q = U_1^T \]  

(32)

In equation (26), \( x \) is hold and more than second power of \( x \) are replaced to \( \hat{x} \) because of the conservation of observation. Projection

![Figure 6: Projection of \( \Xi \) in \( x \)-space on \( \hat{x} \)-subspace](image)

with \( Q \) means that the most probable subspace is calculated to approximate the motion \( \Xi \) as shown in Figure 6. In the reduced controller, the value of terms \( \hat{L} \) is calculated as;

\[ \hat{L} = 1 + n + \sum_{i=2}^{\ell} r H_i = 1 + n + \sum_{i=2}^{\ell} \frac{(r + i - 1)!}{(r - 1)!i!} \]  

(33)

For example, in the case of \( n = 18 \) and \( \ell = 4 \), \( L = 7315 \) (cf. Table 1), the amount of increase of \( \hat{L} \) is smaller than that of \( L \) as shown in Table 2. Measured computational times also gradually decrease when \( r \) becomes smaller by using INTEL Core2 Duo 2.4GHz processor for calculation. Calculation amount of reduced controller is as follows;

\[ \hat{c}(n, \ell, r) = m \left[ \sum_{i=2}^{\ell} \left( \frac{(r + i - 1)!}{(r - 1)!i!} (i + 1) \right) + 2n - 2 \right] \]  

(34)

By using the Stirling’s approximation, the order of \( c(n, \ell, r) \) is calculated as follows;

\[ O^{(c)} \approx O \left( m \sqrt{\frac{r + \ell - 1}{(r - 1)^{\ell}} \frac{(r + i - 1)!}{(r - 1)^{\ell}} (i + 1)} \right) \]  

(35)
Table 2: The value of \( \hat{L} \) and computational time with respect to \( r \) and \( n \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( r )</th>
<th>( \hat{L} )</th>
<th>Time [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>4</td>
<td>8.1</td>
<td>2.2</td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>8.1</td>
<td>4.5</td>
</tr>
<tr>
<td>18</td>
<td>6</td>
<td>8.1</td>
<td>8.1</td>
</tr>
<tr>
<td>18</td>
<td>10</td>
<td>53.2</td>
<td>53.2</td>
</tr>
<tr>
<td>17</td>
<td>10</td>
<td>596.3</td>
<td>53.2</td>
</tr>
<tr>
<td>17</td>
<td>18</td>
<td>731.5</td>
<td>53.2</td>
</tr>
<tr>
<td>15</td>
<td>18</td>
<td>1009</td>
<td>53.2</td>
</tr>
</tbody>
</table>

When \( r \) is smaller than \( n \), the order of \( \hat{c} \) is sufficiently smaller than that of \( c \). This result shows the effectiveness of the proposed reduction method.

It has been already shown that \( \Theta \) is calculated by equation (9), and it requires inverse of \( \Phi \Phi^T \in L_{\times L} \). By the same way, \( \tilde{\Theta} \) is obtained by:

\[
\tilde{\Theta} = U \tilde{\Phi}^# \tag{36}
\]

\[
\tilde{\Phi} = \left[ \tilde{\phi}(x_1, Q) \tilde{\phi}(x_2, Q) \cdots \tilde{\phi}(x_N, Q) \right] \tag{37}
\]

and it requires the calculation of inverse of \( \tilde{\Phi} \tilde{\Phi}^T \in L_{\times L} \), which is much smaller than \( \Phi \Phi^T \). Moreover, the pseudo-reference with the controller reduction requires calculation of pseudo inverse of \( \delta h / \partial x \in L_{\times L} \) which is much smaller than that of \( \delta h / \partial x \in L_{\times L} \) in equation (20).

### 3.2 Validation of controller reduction

#### 3.2.1 Tap dancing robot

The proposed controller reduction is applied to the tap dancing robot which is a simple system for evaluation. The tap dancing robot is shown in Figure 7-(a) and its dynamical model is shown in Figure 7-(b). The detail of the dynamic equation of tap dancing robot is shown in [8]. It steps continuously by changing the stance foot and is stabilized by shaking the head. The input is the torque \( \tau \) and the state variable \( x \) consists of lower body rotational angle \( \theta_1 \), head rotational angle \( \theta_2 \) and their angular velocities \( \dot{\theta}_1, \dot{\theta}_2 \) as follows:

\[
x = \begin{bmatrix} x_p^T & x_v^T \end{bmatrix}^T \in R^4 \tag{38}
\]

\[
x_p = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T, \quad x_v = \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T
\]

Here we assume that the impact of foot on the ground is completely inelastic collision. The referenced motion \( \Xi \) in equation (3) is generated by giving step reference to PD controlled \( \theta_2 \). The robot moves dynamically but unstably. By clipping one cycle motion, \( \Xi \) is obtained and this motion is modeled by the attractor design method.

Because \( n = 4 \) in this robot, \( L \) and \( \hat{L} \) are calculated for different orders of polynomial \( \ell \) as shown in Table 3. \( r = 3 \) is utilized for controller reduction. \( \ell = 2 \) does not stabilize the robot by both original and reduced controller (which is indicated by "failed" in Table 3). This result show that controller reduction method requires only 21 terms which is smaller value than 35 terms to stabilize the robot by using the conventional method. Therefore, \( \ell = 4 \) is used to design the pseudo-reference in the following.

The pseudo-reference is designed by using the conventional controller and the reduced controller. Because \( \hat{x}_e^{ref} \) is not decided uniquely as shown in equation (20), the objective function \( J_1 \) is set as:

\[
J_1 = \left| \left| W_1 (x_p^{ref} - x_p) \right| \right|^2 + \left| \left| W_2 x_v^{ref} \right| \right|^2 \tag{39}
\]

where \( W_1 \) and \( W_2 \) are weighting matrices. The first term aims at making the distance small between \( x_p^{ref} \) and \( x_p \) to avoid a radical variation of \( x_v^{ref} \). The second term aims at making \( x_v^{ref} \) small so that the pseudo-reference shows posture information with small velocity. Figure 8 and 9 show the motion of the tap dancing motion with the conventional controller and the reduced controller respectively. (a) shows the locus of \( x \) with a solid line and the pseudo-reference with dots respectively in \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \) space. (b) shows the motion of the tap dancing robot. Indicated numbers in (b) correspond to those in (a). From these results, we can see that the

![Figure 7: Tap dancing robot[8] and its dynamical model](image)

![Figure 8: Tap dancing motion and pseudo-reference with original controller](image)
The controller is designed by using $\bar{\Theta}(\dot{x}, Q)$. The input $u$ consists of each joint torque as follows;

$$u = \begin{bmatrix} \tau_1 & \tau_2 & \cdots & \tau_6 \end{bmatrix}^T \in R^6 \quad (43)$$

The controller is designed by using $\dot{x}$ as follows;

$$u = \hat{\Theta}(\dot{x}, Q) \quad (44)$$

Human walking motion is measured by an optical motion capture system and it is projected on sagittal plane to obtain $\Xi$ in equation (3). The linear projection $Q$ is obtained using $r = 5$. Table 4 shows relationship between the number of terms in polynomial. The original controller (without reduction) contains 7315 terms with $n = 18$, on the other hand, the reduced controller has only 139 terms by employing $r = 5$.

### 4.2 Pseudo-reference of walking motion

Based on the reduced controller, pseudo-references of walking motion is obtained. To determine $x^{ref}$ in equation (20) uniquely, the following objective function $J_\ell$ is employed and minimized.

$$J_\ell = \left\| W_1(x^{ref}_p[k] - x_p[k]) \right\|^2 + \left\| W_2(x^{ref}_p[k+1] - (x^{ref}_p[k] + T(x^{ref}_v[k])) \right\|^2 \quad (45)$$

where $W_1$ and $W_2$ are weighting matrices and $T$ is sampling time. The first term is same as previous section. The second term aims at satisfaction of dynamic consistency between $x^{ref}_p$ and $x^{ref}_v$. Because of the symmetry of walking motion, the half walking motion in the case of the right supporting leg is represented in Figure 11. The measured walking motion and pseudo-reference are represented by a solid line and a dashed line respectively. The right leg joints are marked by white circles. The posture of pseudo-reference is illustrated so that its hip joint position and orientation coincides to the measured data. From this result, pseudo-reference gives us the following information for human walking.

- In (i), the kicking leg (right leg) moves forward to slow down the body speed.
- In (ii), the swing leg is moved forward strongly for the next gait. The landing leg is stretched to support the weight of the body.
- In (iii), the swing leg is continuously lifted up to prepare for the next foot landing. The ankle of support leg is bended to obtain the propulsion force.

Here we note that

(a) Unfortunately, the obtained reduced controller could not stabilize the legged system. Because $\Xi$ obtained from a motion...
capture system projecting in the sagittal plane, Ξ may not satisfy the dynamic constrain of the planner walking. Transformation of Ξ considering dynamical consistency will be required.

(b) Pseudo-reference obtained from the reduced controller should be compared to original pseudo-reference obtained from the original controller. However, the original controller is not calculated because of the shortage of computational memory because of large number of L.

5 Conclusions

In this paper, we proposed the controller reduction method to design an autonomous controlled system, and pseudo-reference is applied to human motion based on the proposed reduction method. The results of this paper are summarized as follows;

1. Controller reduction of an autonomous control system based on an orbit attractor is proposed by using correlation of motion data.

2. The proposed method is evaluated by the tap dancing robot. The same tap dancing motion and pseudo-reference are realized by using either the proposed controller or the conventional one, which means the effectiveness of the proposed reduction method.

3. The proposed method is also applied to human legged motion which is multi-degree of freedom system. The pseudo-reference showed the significant information to realize the walking motion.

The pseudo-references are designed as movie of postures which contain significant information. In the future, motion instruction for beginner will be performed by using the movie of expert’s pseudo-references.

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