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Subspace Controller Reduction based on Experimental Data

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Abstract

This paper proposes a controller reduction method based on experimental data. This method is applicable to unstable controllers and provides us some indices by which we can determine the reduced order. Furthermore, we evaluate its effectiveness by experiment on an inverted pendulum system.

1 Introduction

So far, many researchers attacked the controller reduction problem and various nice results have been obtained [1, 2]. All of these results were obtained from the view point of control or system approximation. However, as for the system approximation, it seems to be nice to take advantage of the advanced approaches in system identification. In addition, as for controller redesign, it would be reasonable to utilize the closed loop experimental data. Therefore, in this paper, we proposed a new approach to the controller reduction problem based on the Subspace state-space system identification (4SID) method [3], which is known as a powerful tool for identification. Furthermore, we evaluate its effectiveness through experiments.

2 Reduction problem

In this paper, we consider the closed loop system shown in Fig.1, where P is the plant, K is the controller and both r_1 and r_2 are given reference signals. The controller K is represented by time invariant discrete-time state-space equation as

$$K : \begin{cases} x[k+1] = Ax[k] + By[k] \\ u[k] \;\;\;\;\; = Cx[k] + Dy[k] \end{cases}, \quad x[k] \in \boldsymbol{R}^n \tag{1}$$

We assume that the above realization is minimal and that its state $x[k]$ $(k = 1, 2, 3\cdots)$ is available. Our goal is to obtain the reduced order controller K_r given by

$$K_r : \begin{cases} x_r[k+1] = A_r x_r[k] + B_r y[k] \\ u_r[k] \;\;\;\;\; = C_r x_r[k] + Dy[k] \end{cases}, \quad x_r[k] \in \boldsymbol{R}^p \tag{2}$$

which approximately satisfies $u_r[k] \simeq u[k]$ using the closed loop experimental data.

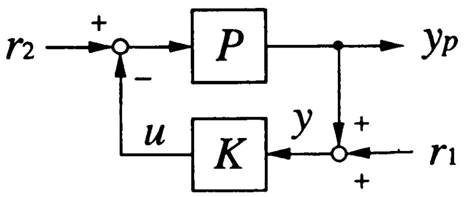

Figure 1: Closed loop system

3 Reduction method

3.1 Basic idea

Define the following matrices.

$$\mathcal{Y}^j := \begin{bmatrix} y[1] & y[2] & \cdots & y[N] \\ \vdots & \vdots & & \vdots \\ y[j] & y[j+1] & \cdots & y[N+j-1] \end{bmatrix} \tag{3}$$

$$X := \begin{bmatrix} x[1] & x[2] & \ldots & x[N+1] \end{bmatrix} \tag{4}$$

$$\Gamma^j := \begin{bmatrix} C \\ \vdots \\ CA^{j-1} \end{bmatrix}, \quad H^j := \begin{bmatrix} D & & 0 \\ \vdots & \ddots & \\ CA^{j-2}B & \cdots & D \end{bmatrix} \tag{5}$$

Similarly, $\mathcal{U}^j$, $\mathcal{U}_r^j$, X_r, Γ_r^j and H_r^j are defined through $u[k]$, $u_r[k]$, $x_r[k]$, and (2). Then, from (1) and (2), we have

$$\mathcal{U}^j - H^j\mathcal{Y}^j = \Gamma^j X, \; \mathcal{U}_r^j - H_r^j\mathcal{Y}^j = \Gamma_r^j X_r \tag{6}$$

Now we consider the situation that

$$\Gamma_r^j X_r \simeq \widehat{\Gamma}^j \widehat{X} \simeq \widehat{\Gamma}^j \left[\frac{\widehat{X}_1}{0}\right] \tag{7}$$

$$\widehat{X} = TX = \left[\frac{\widehat{X}_1}{\widehat{X}_2}\right], \quad \widehat{X}_1 \in \boldsymbol{R}^{p\times N} \tag{8}$$

$$\widehat{\Gamma}^j := \Gamma^j T^{-1} \tag{9}$$

are satisfied. Then, from (6) and (7), the following relations hold.

$$\begin{cases} u[k] \simeq u_r[k] \\ \widehat{C}\widehat{B}y[k] \simeq C_r B_r y[k] \\ \vdots \\ \widehat{C}\widehat{A}^{j-1}\widehat{B}y[k] \simeq C_r A_r^{j-1} B_r y[k] \end{cases}, \quad k = 1, \cdots, N \tag{10}$$

$$\widehat{A} = TAT^{-1}, \widehat{B} = BT^{-1}, \widehat{C} = TC \tag{11}$$

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So we can regard (2) as a reduced order approximation of (1). From the above observation, if we can find a similarity transformation T satisfying (7), we obtain the reduced controller K_r by

$$\hat{A} =: \begin{bmatrix} A_r & * \\ * & * \end{bmatrix},\ \hat{B} =: \begin{bmatrix} B_r \\ * \end{bmatrix},\ \hat{C} =: \begin{bmatrix} C_r & * \end{bmatrix} \tag{12}$$

3.2 Algorithm

First, by using the experimental data of $x[k]$, determine $X \in \boldsymbol{R}^{n \times N}$ in (4). We assume rank$X = n$.

[Step 1] By using A, C, X and a large number j, set Ξ as

$$\Xi := \Gamma X \quad \in \boldsymbol{R}^{jm \times N} \tag{13}$$

[Step 2] Using L_1 calculated by

$$L_1 := \textbf{diag}\{\ \|\xi_1\|^{-1} \quad \cdots \|\xi_{jm}\|^{-1}\ \} \tag{14}$$

$$\Xi = [\ \xi_1^T \quad \cdots \quad \xi_{jm}^T\ ]^T \tag{15}$$

determine Ξ_L by

$$\Xi_L := \boldsymbol{L}_1 \Xi \tag{16}$$

[Step 3] Calculate the singular value decomposition of Ξ_L

$$\begin{aligned} \Xi_L &= [\ E \mid *\ ] \left[\begin{array}{c|c} S & 0 \\ \hline 0 & 0 \end{array}\right] \left[\begin{array}{c} V^T \\ \hline * \end{array}\right] \\ &= ESV^T \\ S &:= \textbf{diag}\{\ \sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_n\ \} \end{aligned} \tag{17}$$

In (17), if we consider

$$\Xi_L = L_1 \hat{\Gamma}^j \hat{X},\ \hat{\Gamma}^j := L_1^{-1} ES,\ \hat{X} := V^T \tag{18}$$

then by using T as

$$X = T\hat{X},\ T := XV \tag{19}$$

and if

$$\sigma_1 \geq \cdots \geq \sigma_p \gg \sigma_{p+1} \geq \cdots \geq \sigma_n \tag{20}$$

is satisfied, it is easy to see that we obtain

$$\Xi_L \simeq \boldsymbol{L}_1 \hat{\Gamma}^j \left[\begin{array}{c} \hat{X}_1 \\ \hline 0 \end{array}\right] \tag{21}$$

So the similarity transformation T given by (19) satisfies (7).

[Step 4] From T in (19), we can obtain the reduced controller K_r by (12).

4 Experiment

4.1 Plant description

In this section, we consider the inverted pendulum system shown in Fig.2. In this system, the angle of the pendulum θ and the arm ϕ are controlled by the input torque τ, and it is a single input and 2 output system.

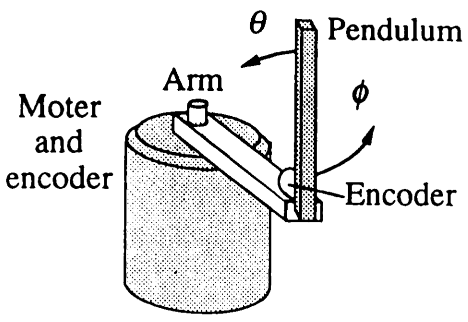

Figure 2: Inverted pendulum system

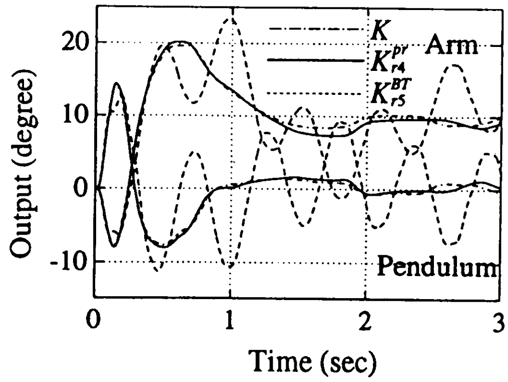

Figure 3: Step responses

4.2 Controller reduction

We design a controller based on the H_∞ loop shaping control[4] which is of the order 12. We transformed $K(s)$ to the discrete time controller $K[z]$ and obtained the 3~7th order reduced controller K_{ri}^{pr}, $(i = 3, 4, ..7)$ by the proposed method (using j=30). The 4~7th order controller could stabilize P. Next, we obtained the 3~7th order reduced controller K_{ri}^{BT}, $(i = 3, 4, ..7)$ by BT method using the normalized coprime factorization. Then, the 5~7th order controller could stabilize P. Fig.3 shows step responses of closed loop systems using K,K_{r4}^{pr} and K_{r5}^{BT}.

References

[1] K. Glover: All Optimal Hankel-Norm Approximations of Linear Multivariable Systems and Their L_∞-error Bounds Int. J. Control, Vol.39, No.6, 1115/1193 (1984)

[2] D. F. Enns: Model Reduction with Balanced Realization : An Error Bound and a Frequency Weighted Generalization, Proc. of 23rd CDC, 127/132 (1984)

[3] M.Verhaegen and P.Dewilde: Subspace Model Identification Part.2 Analysis of the Elementary Output - Error State - Space Model Identification Algorithms; Int. J. Contr., Vol.56, No.5, 1211/1241 (1992)

[4] D. C. McFarlane and K. Glover: Robust Controller Design Using Normalized Coprime Factor Plant Description, Lecture Notes in Control and Information Science, No.138, Springer-Verlag (1990)