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Subspace System Identification considering both Noise Attenuation and Use of Prior Knowledge

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Abstract

This paper proposes a new subspace state-space system identification method which consists of two stages. At the first stage, the noise attenuation is achieved based on the uncorrelation between input signal and noises, where a large amount of data can be handled with the prescribed size matrices. At the second stage, the available prior knowledge on the plant is fully used for the subspace based identification. The combination of these two methods enable us to obtain accurate state-space models. An illustrative numerical example is given to show the effectiveness of the proposed method.

1 Introduction

Recently, a lot of attention has been given to subspace state-space system identification (4SID) method[1]~[7]. In fact, this method can be easily applied to MIMO systems and closed loop identification. However, this method is not so robust against noises and the accurate model is sometimes hard to obtain. One way to circumvent this difficulty is to use large number of experimental data. However it requires more memories and computation power. Therefore, it is necessary to achieve noise reduction with limited computation power. The other major difficulty of 4SID method is that it is not possible to take account of the prior knowledge of the plant such as partial information on pole locations. For example, many mechanical systems have a pole at origin (*i.e.* integral type). If we can take advantage of the prior knowledge, more accurate model would be obtained.

In this paper, we propose a new subspace state-space system identification method taking account of both noise attenuation and use of the prior knowledge. At the first stage, we attenuate the noises in the input-output data based on the uncorrelation between input signal and noise, where large amount of data can be handled with the prescribed size matrices. At the second stage, as a prior knowledge on the plant, the pole location is assumed to be known partly. We show how to make use of the prior knowledge for subspace identification. The combination of these two methods enable us to obtain accurate state-space models. In order to illustrate the effectiveness of the proposed method, we give numerical examples.

In this paper, $[\cdot]^{\dagger}$ means the generalized inverse matrix, $\|\cdot\|$ means the Frobenius norm. And $A(i:j, k:\ell)$ represents the submatrix of A consisting of block rows $i-j$ and block columns $k-\ell$. A/B means the orthogonal projection of A to the column space of B, which is given as

$$A/B = AB^T(BB^T)^{\dagger}B$$

2 System description

Consider the following linear, discrete-time SIMO system.

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + Du_k + v_k \end{aligned} \tag{1}$$

Here $y \in \boldsymbol{R}^{\ell}$ is the output signal, $u \in \boldsymbol{R}$ is the input signal, $x \in \boldsymbol{R}^n$ is the state vector, $w \in \boldsymbol{R}^n$ and $v \in \boldsymbol{R}^{\ell}$ are unknown zero mean process noise and additive noise, respectively. And we assume that

- w, v and the input u uncorrelate to each other, namely,

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} w_{i+k} u_k^T = 0 \tag{2}$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} v_{i+k} u_k^T = 0 \tag{3}$$

  are satisfied for any i.

- This system is minimal and we know the pole location of the plant partly, by which the system in (1) can be written as

$$\begin{aligned} \begin{bmatrix} x_{k+1}^k \\ \xi_{k+1} \end{bmatrix} &= \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_k^k \\ \xi_k \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ w_{2\cdot k} \end{bmatrix} \\ y_k &= \begin{bmatrix} C_1 C_2 \end{bmatrix} \begin{bmatrix} x_k^k \\ \xi_k \end{bmatrix} + Du_k + v_k \end{aligned} \tag{4}$$

  where $w_2 \in \boldsymbol{R}^{n-p}$ is a noise and A_{11} and B_1 are known matrices, and $x_k^k \in \boldsymbol{R}^p$ is available. Without loss of generality, we assume that the

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pair (A_{11}, B_1) has the following canonical form.

$$A_{11} = \begin{bmatrix} a_1 & 1 & & 0 \\ \vdots & \vdots & \ddots & \\ a_{p-1} & 0 & \cdots & 1 \\ a_p & 0 & \cdots & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (5)$$

Our goal is to identify the coefficient matrices $(A_{21}, A_{22}, B_2, C_1, C_2, D)$ of (4) based on the subspace based system identification method.

3 4SID method

In this section we briefly review 4SID method[5].

3.1 Input output equation

The existing 4SID method obtains the coefficient matrices (A, B, C, D) directly from input-output data. From (1), sequences u, y, x satisfy the following general structured input-output equation.

$$Y_{i,j,N} = \Gamma_i X_j^d + H_i U_{i,j,N} + \Gamma_i X_j^s + V_{i,j,N} \quad (6)$$

where,

$$Y_{i,j,N} := \begin{bmatrix} y_j & \cdots & y_{j+N-1} \\ \vdots & & \vdots \\ y_{j+i-1} & \cdots & y_{j+i+N-1} \end{bmatrix} \quad (7)$$

$$X_j^d := [\ x_j^d \ \cdots \ x_{j+N-1}^d \ ] \quad (8)$$

$$X_j^s := [\ x_j^s \ \cdots \ x_{j+N-1}^s \ ] \quad (9)$$

$$\Gamma_i := \begin{bmatrix} C \\ \vdots \\ CA^{i-1} \end{bmatrix}, \quad H_i := \begin{bmatrix} D & & 0 \\ \vdots & \ddots & \\ CA^{i-2}B & \cdots & D \end{bmatrix} \quad (10)$$

$U_{i,j,N}$ and $V_{i,j,N}$ are defined in the same way as $Y_{i,j,N}$ by u and v, respectively. The state vector x is split in the deterministic part x^d and stochastic part x^s as

$$x_{k+1}^d = Ax_k^d + Bu_k, \quad x_{k+1}^s = Ax_k^s + w_k \quad (11)$$

Γ_i is the extended observablity matrix, and H_i is a Toeplitz matrix.

3.2 Identification

4SID method is based on (6). By the orthogonal projection of $Y_{i,j,N}$ to the column space of $U_{i,j,N}$ and $U_{i,j,N}^{\perp}$, $Y_{i,j,N}$ is decomposed as follows

$$Y_{i,j,N} = Y_{i,j,N}/U_{i,j,N}^{\perp} + Y_{i,j,N}/U_{i,j,N} \quad (12)$$

$$Y_{i,j,N}/U_{i,j,N}^{\perp} = \Gamma_i X_i^d/U_{i,j,N}^{\perp} + S_{i,j,N}/U_{i,j,N}^{\perp} \quad (13)$$

$$Y_{i,j,N}/U_{i,j,N} = \Gamma_i X_i^d/U_{i,j,N} + H_i U_{i,j,N} + S_{i,j,N}/U_{i,j,N} \quad (14)$$

$$S_{i,j,N} := \Gamma_i X_i^s + V_{i,j,N} \quad (15)$$

When noises are small,

$$\|\Gamma_i X_i^d/U_{i,j,N}^{\perp}\| \gg \|S_{i,j,N}/U_{i,j,N}^{\perp}\| \quad (16)$$

is satisfied. Because Γ_i is the extended observability matrix, $\text{rank}\Gamma_i = n$ and

$$\text{rank} Y_{i,j,N}/U_{i,j,N}^{\perp} = n \quad (17)$$

is satisfied. By using these relations, we can identify Γ_i and C, A from (13), and by using Γ_i, B and D are obtained based on (14). The algorithm of 4SID method is shown in Appendix.

However, when noises are not small, as (16) is not satisfied, Γ_i is not obtained accurately. In this case, we need to attenuate the noise term $\Gamma_i X_i^s + W_{i,j,N}$ from (6) in advance. Furthermore, the existing 4SID method is not possible to take account of the prior knowledge of the plant. In general case, we often have a partial information on pole location of the plant. For example, many mechanical systems have a pole at origin. In the following, we propose a new subspace state-space system identification method taking account of both the noise attenuation and use of the prior knowledge.

4 Proposed method

Our method consists of two stages. The first stage is the noise attenuation. The second stage is using prior knowledge.

4.1 Stage 1 : Noise attenuation

In this section, we show how to attenuate the noise term in (6) by using the uncorrelation of the input signal and noises, where large amount of data can be handled with the prescribed size matrices.

4.1.1 Design Concept:

Rewrite (6) as

$$Y_{i,\alpha+1,N} = \Gamma_i X_{\alpha+1}^d + H_i U_{i,\alpha+1,N} + S_{i,\alpha+1,N} \quad (18)$$

where α is user-defined index. Multiply (18) by $\frac{1}{N} U_{\alpha+i-1,1,N}^T$ from the right, then we have

$$\frac{1}{N} Y_{i,\alpha+1,N} U_{\alpha+i-1,1,N}^T = \Gamma_i \frac{1}{N} X_{\alpha+1}^d U_{\alpha+i-1,1,N}^T + H_i \frac{1}{N} U_{i,\alpha+1,N} U_{\alpha+i-1,1,N}^T + \frac{1}{N} S_{i,\alpha+1,N} U_{\alpha+i-1,1,N}^T \quad (19)$$

Because u uncorrelates with both v and w,

$$\lim_{N\to\infty} \left\| \frac{1}{N} S_{i,\alpha+1,N} U_{\alpha+i-1,1,N}^T \right\| = 0 \quad (20)$$

is satisfied. And because u correlates with x^d and y,

$$\lim_{N\to\infty} \left\| \frac{1}{N} X_{\alpha+1}^d U_{\alpha+i-1,1,N}^T \right\| \neq 0 \quad (21)$$

$$\lim_{N\to\infty} \left\| \frac{1}{N} Y_{i,\alpha+1,N} U_{\alpha+i-1,1,N}^T \right\| \neq 0 \quad (22)$$

hold. It is obvious

$$\lim_{N\to\infty} \left\| \frac{1}{N} U_{i,\alpha+1,N} U_{\alpha+i-1,1,N}^T \right\| \neq 0 \quad (23)$$

From these observation, when $N \to \infty$, (19) is written as

$$\hat{Y} = \Gamma_i \hat{X} + H_i \hat{U}, \quad \hat{Y} := \frac{1}{N} Y_{i,\alpha+1,N} U^T_{\alpha+i-1,1,N} \quad (24)$$

$$\hat{X} := \frac{1}{N} X^d_{\alpha+1} U^T_{\alpha+i-1,1,N}, \quad \hat{U} := \frac{1}{N} U_{i,\alpha+1,N} U^T_{\alpha+i-1,1,N} \quad (25)$$

We can regard (24) as the input-output equation achieving the noise attenuation. This stage does not need the assumption that this system is SIMO system. *i.e.* this stage can be adopted to MIMO systems.

4.1.2 Discussions:

Reduction of the matrix size: In the stage 1, by multipling (18) by $\frac{1}{N} U^T_{\alpha+i-1,1,N}$, noise attenuation is achieved because of (20), (22) and (23). At the same time, large amount of data can be handled with the prescribed size matrix. In the conventional 4SID method, the input-output equation (6) is $\ell i \times N$ matrix. Because we must compute QR decomposition such as (A.1) using this size of matrix, if $N \to \infty$ then quite a large amount of computation must be required. However, in the proposed method, input output equation is given by (24) whose matrix size is $\ell i \times (\alpha+1)$ $(\alpha \ll N)$. Because α is a design parameter and it does not depend on N, our method requires much less computation than the existing 4SID method.

Impulse response of the system: It is useful to know the impulse response of the system. In our method, by using the zero mean stationary white signal as the input, the impulse response of the system is obtained easily.

Consider when u is zero mean white signal with covariance σ_u^2 and $x_1 = 0$. Because the output $y_{\alpha+1}$ is written as

$$y_{\alpha+1} = Du_{\alpha+1} + \sum_{k=1}^{\alpha} CA^{k-1} B u_{\alpha-k+1} + v_{\alpha+1} + \sum_{k=1}^{\alpha} CA^{k-1} w_{\alpha-k+1} \quad (26)$$

the first ℓ column and first row of $\frac{1}{N} Y_{i,\alpha+1,N} U^T_{\alpha+i-1,1,N}$ is written as

$$\frac{1}{N} \begin{bmatrix} y_{\alpha+1} & y_{\alpha+2} & \cdots & y_{\alpha+N} \end{bmatrix} \begin{bmatrix} u_1 & u_2 & \cdots & u_N \end{bmatrix}^T = CA^{\alpha-1} B \sigma_u^2 \quad (N \to \infty) \quad (27)$$

where we use the uncorrelation of u_k and v_k, w_k, u_ℓ $(\ell \neq k)$. By extending this observation to all columns and rows,

$$\lim_{N\to\infty} \frac{1}{N} Y_{i,\alpha+1,N} U^T_{\alpha+i-1,1,N} = \sigma_u^2 \begin{bmatrix} CA^{\alpha-1}B \cdots & D & & 0 \\ CA^{\alpha}B & \cdots CB & D & \\ \vdots & \vdots & \vdots & \ddots \\ CA^{\alpha+i-2}B \cdots & CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix} \quad (28)$$

is satisfied. This matrix becomes a Toeplitz matrix consisting of the impulse response of the system.

4.2 Stage 2 : Subspace based identification using prior knowledge

We often have a partial information on pole location of the plant. In this section, we show how to use this information.

4.2.1 Basic idea: The input-output equation of the system in (4) is given by

$$Y_{i,j,N} = \bar{\Gamma}_i \Xi_j + H_i^1 X_j^k + H_i^2 U_{i,j,N} + S_{i,j,N} \quad (29)$$

where,

$$\Xi_j := \begin{bmatrix} \xi_j & \cdots & \xi_{j+N-1} \end{bmatrix} \quad (30)$$

$$X_j^k := \begin{bmatrix} x_j^k & \cdots & x_{j+N-1}^k \end{bmatrix} \quad (31)$$

$$H_i^1 := \begin{bmatrix} \Sigma_1^T & \cdots & \Sigma_i^T \end{bmatrix} \quad (32)$$

$$\Sigma_k := \begin{cases} C_1 & (k=1) \\ \Sigma_{k-1} A_{11} + C_2 A_{22}^{k-2} A_{21} & (k \geq 2) \end{cases} \quad (33)$$

$$\bar{\Gamma}_i := \begin{bmatrix} C_2 \\ \vdots \\ C_2 A_{22}^{i-1} \end{bmatrix} \quad (34)$$

and H_i^2 is a block Toeplitz matrix with the first column

$$H_i^2(1:i\ell,:) = \begin{bmatrix} D \\ \Sigma_1 B_1 + C_2 B_2 \\ \vdots \\ \Sigma_{i-1} B_1 + C_2 A_{22}^{i-1} B_2 \end{bmatrix} \quad (35)$$

Moreover, the following equations are satisfied.

$$H_i^1(\ell+1:i\ell,:) = H_i^1(1:(i-1)\ell,:) A_{11} + \bar{\Gamma}_i(1:(i-1)\ell,:) A_{21} \quad (36)$$

$$H_i^2 = \Lambda + \widehat{H}_i^2 \quad (37)$$

Here,

$$\widehat{H}_i^2 := \begin{bmatrix} D & & 0 \\ C_2 B_2 & D & \\ \vdots & \vdots & \ddots \\ C_2 A_{22}^{i-2} B_2 & C_2 A_{22}^{i-3} B_2 & \cdots & D \end{bmatrix} \quad (38)$$

and Λ is a block Toeplitz matrix with the first column

$$\Lambda(1:i\ell,:) = \begin{bmatrix} 0 \\ H_i^1(1:(i-1)\ell,:) B_1 \end{bmatrix} \quad (39)$$

By using these relations, we identify unknown parameters $(A_{21}, A_{22}, B_2, C_1, C_2, D)$.

4.2.2 Discussions: In the existing 4SID method, because the obtained state space has no meaning in physical sense, we can not use the prior knowledge even if we know the pole location of the system, partly. It is expected that if we can use this information, more accurate models are obtained. On the other hand, in our method, we can use the prior knowledge. Moreover, we do not use the knowledge of the zero location of the system. If we try to split the system P to the known part P_k and the unknown part P_{un} such that

$$P = P_{un} + P_k \quad \text{or} \quad P = P_{un} \cdot P_k \tag{40}$$

we must require the zero location of P_k. However in our method, we need only the partial pole location.

In the stage 1, we consider the system shown in (1) whose input-output equation is (6). On the other hand, the stage 2 considers the system shown in (4) whose input-output equation is (29). However, the noise attenuation in the stage 1 can be adopted to (29) and we can combine the noise attenuation and use of the prior knowledge. The algorithm of the method in the stage 2 is shown in the next section as the combined method with the stage 1.

4.3 Algorithm

In this section, we show the algorithm of our method. At first, rewrite (29) as

$$Y_{i,\alpha+1,N} = \bar{\Gamma}_i \Xi_{\alpha+1} + H_i^1 X_{\alpha+1}^k + H_i^2 U_{i,\alpha+1,N} + S_{i,\alpha+1,N} \tag{41}$$

where $X_{\alpha+1}^d$ is obtained from a simulation.

【Step1】 Noise attenuation: Multiply (41) by $\frac{1}{N} U_{\alpha+i-1,1,N}^T$ from the right, we have the following input-output equation achieving the noise attenuation.

$$\widehat{Y} = \bar{\Gamma}_i \widehat{\Xi} + H_i^1 \widehat{X} + H_i^2 \widehat{U} \tag{42}$$

$$\widehat{Y} := \frac{1}{N} Y_{i,\alpha+1,N} U_{\alpha+i-1,N}^T, \quad \widehat{\Xi} := \frac{1}{N} \Xi_{\alpha+1} U_{\alpha+i-1,N}^T \tag{43}$$

$$\widehat{X} := \frac{1}{N} X_{\alpha+1}^k U_{\alpha+i-1,N}^T, \quad \widehat{U} := \frac{1}{N} U_{i,\alpha+1,N} U_{\alpha+i-1,N}^T \tag{44}$$

$$\left(\lim_{N \to \infty} \frac{1}{N} \| S_{i,\alpha+1,N} U_{\alpha+i-1,N}^T \| \to 0 \right)$$

Here we assume

$$\text{rank} \begin{bmatrix} \widehat{X} \\ \widehat{U} \end{bmatrix} = p + i \tag{45}$$

【Step2】 QR decomposition: By using $\widehat{Y}$, $\widehat{U}$ and $\widehat{X}$, compute the following QR decomposition.

$$\begin{bmatrix} \widehat{X} \\ \widehat{U} \\ \widehat{Y} \end{bmatrix} = \begin{bmatrix} R_{11} & & 0 \\ R_{21} & R_{22} & \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \tag{46}$$

From (46), following equations hold.

$$\widehat{Y} = \widehat{R}_{3(:,1:p)} \widehat{X} + \widehat{R}_{3(:,p+1:p+i)} \widehat{U} + R_{33} Q_3 \tag{47}$$

$$\widehat{R}_3 := \begin{bmatrix} R_{31} & R_{32} \end{bmatrix} \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix}^{-1} \tag{48}$$

【Step3】 Identification of C_2, A_{22}: Because $S_{i,\alpha+1,N}$ is attenuated,

$$R_{33} Q_3 = \bar{\Gamma}_i \widehat{\Xi} \Big/ \begin{bmatrix} \widehat{X} \\ \widehat{U} \end{bmatrix}^{\perp} \tag{49}$$

is satisfied from (29) and (47). By the singular value decomposition of R_{33}

$$R_{33} = \begin{bmatrix} U_n & U_n^{\perp} \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \tag{50}$$

$$S_1 := \text{diag}\{ \sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_{n-p} \} \tag{51}$$

$$S_2 := \text{diag}\{ \sigma_{n-p+1} \quad \sigma_{n-p+2} \quad \cdots \quad \sigma_{i\ell} \} \tag{52}$$

$\bar{\Gamma}_i$ is obtained by

$$\bar{\Gamma}_i = U_n \tag{53}$$

where we assume $\sigma_{n-p} \gg \sigma_{n-p+1}$. From (53), C_2 and A_{22} are obtained by

$$C_2 = U_{n(1:\ell,:)} \tag{54}$$

$$A_{22} = \left[U_{n(1:(i-1)\ell,:)} \right]^{\dagger} U_{n(\ell+1:i\ell,:)} \tag{55}$$

【Step4】 Identification of C_1, A_{21}: From (42) and (47), H_i^1 is given as

$$H_i^1 = \widehat{R}_{3(:,1:p)} \tag{56}$$

and from (32), (36) and (53), by using (56), C_1 and A_{21} are given as

$$C_1 = H_i^1(1:\ell,:) \tag{57}$$

$$\begin{aligned} A_{21} = {} & \left[U_{n(1:(i-1)\ell,:)} \right]^{\dagger} \\ & \times \left[H_i^1(\ell+1:i\ell,:) - H_i^1(1:(i-1)\ell,:) A_{11} \right] \end{aligned} \tag{58}$$

【Step5】 Identification of B_2, D: From (37),

$$\begin{aligned} \left(U_n^{\perp}\right)^T \widehat{H}_i^2 &= \left(U_n^{\perp}\right)^T \left(H_i^2 - \Lambda\right) \\ &= \left(U_n^{\perp}\right)^T \left[\widehat{R}_{3(:,p\ell+1:(p+i)\ell)} - \Lambda \right] \end{aligned} \tag{59}$$

is satisfied. Because Λ is obtained from (39), B_2 and D are obtained based on the following equation.

$$\begin{aligned} \begin{bmatrix} L(:,1) \\ L(:,2) \\ \vdots \\ L(:,i) \end{bmatrix} = {} & \begin{bmatrix} U_n^{\perp}(1:\ell,:)^T & \cdots U_n^{\perp}((i-1)\ell+1:i\ell,:)^T \\ U_n^{\perp}(\ell+1:2\ell,:)^T & 0 \\ \vdots & \vdots \\ U_n^{\perp}((i-1)\ell+1:i\ell,:)^T \cdots & 0 \end{bmatrix} \\ & \times \begin{bmatrix} I_\ell & 0 \\ 0 & U_n(1:(i-1)\ell,:) \end{bmatrix} \begin{bmatrix} D \\ B \end{bmatrix} \end{aligned} \tag{60}$$

$$L := (U_n^{\perp})^T \left(\widehat{R}_3(:, p\ell+1, (p+i)\ell) - \Lambda \right) \tag{61}$$

5 Numerical example

In this section, we evaluate the effectiveness of the proposed method by numerical example with the closed loop identification.

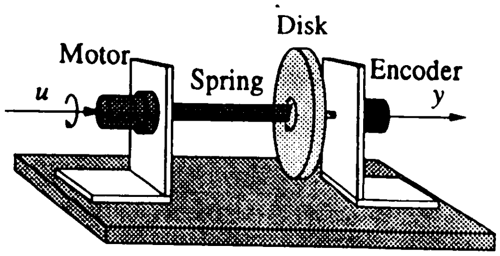

Figure 1: 2-mass spring system

5.1 System description

Consider the 2-mass spring system shown in Fig.1. The output y is the disk rotation and the input is the torque of motor. By considering the equation of motion, the nominal transfer function $P_m(s)$ of this system is given by[8]

$$P_m(s) = \frac{2.38 \times 10^5}{s(s+13.6)(s+a)(s+\bar{a})}, \quad a = 8.15 + 32.1j \tag{62}$$

While we assume that the transfer function $P(s)$ of the real system is given by

$$P(s) = \frac{1.19 \times 10^5}{s(s+15.2)(s+b)(s+\bar{b})}, \quad b = 8.95 + 48.6j \tag{63}$$

Because this system has one inherent integrator and the pole at $s = -13.6$ is perturbed a little, we regard this system has inherent poles at $s = 0, -13.6$. By using this information, we identify $P(s)$ in the discrete-time domain using the closed loop identification.

5.2 Closed loop identification

Consider the closed loop system shown in Fig.2. Here, K is the controller designed by the H_∞ loop shaping control[9] using the model $P_m(s)$ with appropriate weighting function, w and v are noises and r is the reference signal which is M-sequence signal. From y, u and x^k, we compute $\widehat{Y}$, $\widehat{U}$ and $\widehat{X}^k$ defined in (43) and (44) using $i = 10$, $\alpha = 250$, $N = 16000$ and the sampling time $T = 0.01(s)$, and identify (A_{21}, A_{22}, B_2, C_1, C_2, D). In the closed loop identification, it is known that the input u has correlation with noises, and (20) is not satisfied. However, by the same consideration as in section 4.1, we can make use of the noise reduction scheme as follows.

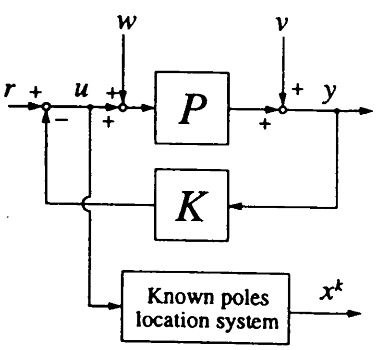

Figure 2: Closed loop system

Split the basic equation (29) into a deterministic and stochastic part as follows,

$$\begin{aligned} Y^d_{i,j,N} + Y^s_{i,j,N} &= \bar{\Gamma}_i(\Xi^d_j + \Xi^s_j) \\ &+ H^1_i(X^{kd}_j + X^{ks}_j) + H^2_i(U^d_{i,j,N} + U^s_{i,j,N}) \end{aligned} \tag{64}$$

where the superscript "d" means the *deterministic* part which is the part corresponding to the refernce signal r, and "s" implies the *stochastic* part which corresponds to the noise. Because r does not correlate with the noise but the deterministic term, if we use such $R_{\alpha+i-1,1,N}$ in place of $U_{\alpha+i-1,1,N}$ in (43), (44) as

$$R_{\alpha+i-1,1,N} := \begin{bmatrix} r_1 & r_2 & \cdots & r_N \\ \vdots & \vdots & & \vdots \\ r_{\alpha+i-1} & r_{\alpha+i} & \cdots & r_{\alpha+i+N-2} \end{bmatrix} \tag{65}$$

then

$$\lim_{N\to\infty} \left\| \frac{1}{N} [\cdot]^d R^T_{\alpha+i-1,1,N} \right\| \neq 0 \tag{66}$$

$$\lim_{N\to\infty} \left\| \frac{1}{N} [\cdot]^s R^T_{\alpha+i-1,1,N} \right\| = 0 \tag{67}$$

hold. Therefore noise attenuation can be achieved in the same way in section 4.1. The stage 2 can be performed as shown in section 4.2.

The gain plots of the identified model $P^{pr}_m(s)$ is shown in Fig.3. For comparison, we identify another model P^{co}_m by using the conventional 4SID method[2]. Both models are identified as 4th order system (in the proposed method, two known poles and two unknown poles). It is clear that P^{co}_m can not be identified accurately in the lower and the higher frequency. Moreover for the model validation, we re-design the controller K^{pr} based on $P^{pr}_m(s)$ and K^{co} based on P^{co}_m using the same weighting function as before. The step responses of $G(P,K)$, $G(P,K^{pr})$, $G(P^{pr}_m, K^{pr})$ and $G(P,K^{co})$ are shown in Fig.4, where

$$G(\mathcal{P},\mathcal{K}) := \frac{\mathcal{PK}}{I+\mathcal{PK}} \tag{68}$$

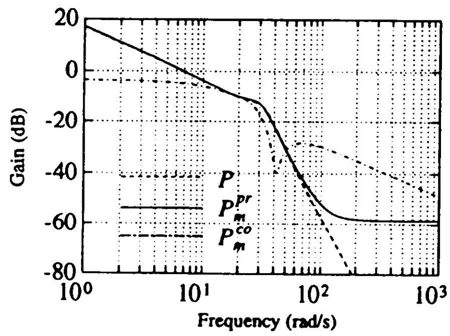

Figure 3: Gain plots

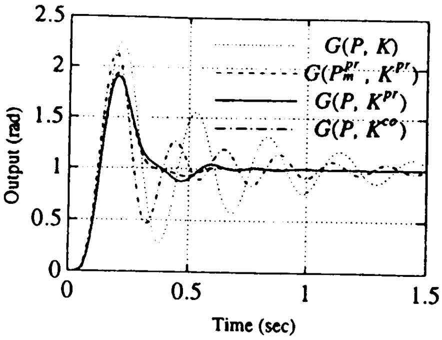

Figure 4: Step responses

The step response of $G(P, K^{pr})$ is similar to that of $G(P_m^{pr}, K^{pr})$ while $G(P, K^{co})$ is not. These results tell us that P_m^{pr} represents P more accurately than P_m^{co}, which shows the effectiveness of our method.

6 Conclusions

In this paper, we have proposed a new subspace state-space system identification method taking account of both the noise attenuation and use of the prior knowledge. At the first stage, we have attenuated the noises in the input-output data based on the uncorrelation between the input signal and the noise, where large amount of data can be handled with the prescribed size matrices. At the second stage, as a prior knowledge on the plant, the pole location have been assumed to be known partly. We have shown how to make use of the prior knowledge for subspace identification. Furthermore, the effectiveness of our method have been illustrated by numerical examples.

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Appendix

A Algorithm of 4SID method

【Step1】: By using $Y_{i,j,N}$ and $U_{i,j,N}$, calculate QR decomposition such as

$$\begin{bmatrix} U_{i,j,N} \\ Y_{i,j,N} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tag{A.1}$$

when, the projections shown in (13), (14) is given as

$$Y_{i,j,N}/U_{i,j,N} = R_{21}R_{11}^{-1}U_{i,j,N} \tag{A.2}$$

$$Y_{i,j,N}/U_{i,j,N}^{\perp} = R_{22}Q_2 \tag{A.3}$$

【Step2】: By using the singular value decomposition of R_{22}

$$R_{22} = \begin{bmatrix} U_n & U_n^{\perp} \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \tag{A.4}$$

$$S_1 := \mathrm{diag}\{\sigma_1 \ \ \sigma_2 \ \cdots \ \sigma_n\} \tag{A.5}$$

$$S_2 := \mathrm{diag}\{\sigma_{n+1} \ \ \sigma_{n+1} \ \cdots \ \sigma_{i\ell}\} \tag{A.6}$$

if $\sigma_n \gg \sigma_{n+1}$ is satisfied, we can regard Γ_i as

$$U_n = \Gamma_i \tag{A.7}$$

because of (17). From (A.7), C and A are obtained by

$$C = U_n(1:\ell,:) \tag{A.8}$$

$$A = [U_n(1:(i-1)\ell,:)]^{\dagger}\, U_n(\ell+1:i\ell,:) \tag{A.9}$$

【Step3】: Because

$$(U_n^{\perp})^T R_{21}R_{11}^{-1} = (U_n^{\perp})^T H \tag{A.10}$$

is satisfied from (14) and (A.2), by determining Ξ as

$$\Xi := (U_n^{\perp})^T R_{21}R_{11}^{-1} \tag{A.11}$$

B and D are obtained from the following relation.

$$\begin{bmatrix} \Xi(:,1) \\ \vdots \\ \Xi(:,i) \end{bmatrix} = \begin{bmatrix} U_n^{\perp}(1:\ell,:)^T & \cdots U_n^{\perp}((i-1)\ell+1:i\ell,:)^T \\ \vdots & \vdots \\ U_n^{\perp}((i-1)\ell+1:i\ell,:)^T \cdots & 0 \end{bmatrix} \times \begin{bmatrix} I_\ell & 0 \\ 0 & U_n(1:(i-1)\ell,:) \end{bmatrix} \begin{bmatrix} D \\ B \end{bmatrix} \tag{A.12}$$

where I_ℓ is ℓth order identity.